# Calibrating Receiver Frequency and Soundcard Sampling Rate using an Off-Air Standard Frequency

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## 1) Introduction

When using a synthesised receiver and PC with sound card with weak-signal software such as WOLF [http://freenet-homepage.de/dl4yhf/wolf/index.html], or to make high-accuracy measurements of signal frequencies, it is necessary to calibrate the receiver frequency and sound card sample rate. Off-air frequency standards are readily accessible, such as MSF on 60kHz and the Droitwich 198kHz transmitter. But the frequency measured by the RX/sound-card combination is affected both by receiver frequency errors and sound card sampling rate errors, so it is not possible with a single measurement to separate and calibrate both sources of error. The sound card sample rate can be easily calibrated if a precision audio-frequency source is available, and the calibrated sound card used in conjunction with the off-air signal to determine the receiver error, but many amateur shacks do not contain a suitable audio source. An elegant way around this problem using the receiver in AM mode to receive Loran-C pulses has been developed by John Andrews, W1TAG [http://www.w1tag.com/WOLFSamp.htm]. An alternative method that simultaneously calibrates both sample rate and receiver error, using the receiver in SSB mode and any convenient LF standard frequency is given below.

The basic idea is that making two frequency measurements allows simultaneous equations to be constructed and solved that determine both the RX frequency error and sampling rate error "unknowns". There are several ways this could be done; e.g. by measuring two different off-air signals, by setting the receiver to produce different audio output frequencies from the same signal. But a convenient method with most receivers is to measure the audio frequency output produced by the same signal in USB and LSB modes. For example, if the receiver frequency is set 1.5kHz below the signal frequency in USB mode, and 1.5kHz above signal frequency in LSB mode, then with no errors the measured audio pitch will be 1.5kHz exactly in both cases. But if the actual RX LO frequency is low, the output pitch on USB will be higher, while the LSB pitch will be lower and vice-versa. However, if the RX frequency is correct, any errors measured will be due to the sound card sample rate, and will be the same irrespective of whether USB or LSB is used. In this case, a low sample rate will lead to higher apparent audio pitch and vice versa. At an intuitive level, the receiver frequency error is about half the *difference* between the audio pitches measured in USB and LSB, while the error produced by the sound card is roughly the difference between the expected audio pitch and the *average* of the measured USB and LSB pitch. In this way, errors due to receiver frequency and sound card sample rate can be separated out.

Section 2 gives the method and formulae used to perform calibration. Before using it, however, read Section 3 on the limitations of this method. If you are interested to know how the formulae were derived, see Section 4.

## 2) Calibration Method and Example

Select a suitable LF standard frequency signal – this should have known accurate frequency, a good signal-to-noise ratio and a strong carrier component – examples in Europe are standard frequency stations like MSF (60kHz), HBG (75kHz), DCF77 (77.5kHz), and some LW broadcasters such as DLF (153kHz) Allouis (162kHz), Droitwich (198kHz). 198kHz is used as an example here, and DL4YHF's "WOLF GUI" software assumed. The example numbers quoted are the actual figures obtained using an FT-817 as the receiver, and an old 300MHz laptop PC.

Set the receiver to USB mode and a carrier frequency 1.5kHz below the standard frequency (196.500kHz). In the WOLF configuration screen, set the "receive options" sample rate to the nominal value (8000Hz) and the centre frequency to 1500Hz. From the "mode" menu select "frequency measurement". Check that the signal carrier is centred in the WOLF spectrum display. After 96 seconds you should have something like:

```
2009-09-30 16:38:07 >WOLF10  -r 8000.000 -f 1500 -m 96 -w 0.0000
t:  96 f: 0.198 a: 0.5 dp: 18.7
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"f: 0.198" is the frequency offset measurement. In this case, Wolf measured a frequency offset from the nominal 1500Hz of +0.198Hz. Call this offset $f_{UO}$. Now set the RX to LSB, set the carrier frequency to 1.5kHz above the standard frequency (199.500kHz), and repeat the WOLF frequency measurement. For this example, the frequency offset was now +0.060Hz. Call this offset $f_{LO}$. Now the sample rate and receiver frequency errors can be calculated:

$f_{STD}$ = frequency of the off-air standard, e.g. 198000Hz
$f_A$ = expected audio frequency, e.g. 1500Hz
$f_{UO}$ = Measured offset in USB mode, e.g. +0.198Hz
$f_{LO}$ = Measured offset in LSB mode, e.g. +0.060Hz
$f_s$ = nominal sample rate, e.g 8000Hz
$f_s'$=actual sample rate
$\Delta f_r / f_r$ = fractional error in the receiver carrier frequency

The actual sample rate is:

$$f_s' = \frac{f_s}{1 + \left(\frac{f_{UO} + f_{LO}}{2 f_A}\right) + \left(\frac{f_{UO} - f_{LO}}{2 f_{STD}}\right)}$$

= 7999.309 samples/sec in the example

The fractional error in the receiver frequency is:

$$\frac{\Delta f_r}{f_r} = \left(\frac{f_{LO} - f_{UO}}{2 f_A + f_{UO} + f_{LO}}\right)\left(\frac{f_A}{f_{STD}}\right)$$

= -3.48 x $10^{-7}$, or - 0.348 parts per million (quite good…)

The actual sample rate value can now be entered into the sample rate boxes in the WOLF configuration screen. Calibrating the sample rate using a precision audio source yielded a value of 7999.328 samples/sec; the results agree within about 2 parts per million. In normal use, the actual signal frequency will be different to the standard frequency used for calibration. For WOLF reception, we would like to configure the software with an accurate value for the expected audio frequency of the signal being processed by the sound card. Suppose we wish to receive a WOLF signal at 503.5kHz, and we set the receiver to 502.5kHz USB to obtain a nominal 1kHz audio output. The actual audio output frequency can be calculated from:

$f_{Audio}$ = actual audio frequency of received signal
$f_{SIG}$ = carrier frequency of signal, e.g. 503500 Hz
$f_{Carrier}$ = receive frequency setting in USB mode, e.g. 502500 Hz

$$f_{Audio} = f_{SIG} - f_{Carrier} - \frac{\Delta f_r}{f_r} f_{Carrier}$$

= 1000.175 Hz, in this case close enough to 1000Hz

When doing these calculations, take care to include any minus signs and powers of 10; also, make sure all frequencies are in hertz, not kilohertz!

The method and formulae above are intended for use with WOLF, however the same technique could be used with other sound card based software such as Spectrum Lab, Argo and others. In this case, bear in mind that the audio frequency measurements will be of the actual frequency, rather than the offset from the expected frequency as measured by WOLF. So if similar measurements to those above were done using Argo, for example, on USB you might measure a frequency of 1500.20 Hz, and on LSB

1499.75Hz. Then $f_{UO}$ = 1500.20 – 1500 = +0.20Hz, and $f_{LO}$ = 1499.75 – 1500 = -0.25Hz. To get worthwhile accuracy in the sample rate calibration, resolution of about 0.01Hz in the frequency measurement is required. This means using spectrogram settings for "QRSS120" or similar. Be aware that the sample rate calibration will have to be repeated for each different sample rate used.

It is not necessary to use an audio frequency of 1500Hz; higher audio frequencies will improve accuracy of sample rate calibration (and lower frequencies make it less accurate). Just enter the appropriate value of expected audio frequency, $f_A$, into the formulae and WOLF configuration, and set the receiver frequencies accordingly. Most receivers display carrier frequency in SSB mode, but some (e.g. selective level meters) display the centre frequency of the receiver passband instead, as do receivers in CW mode. This needs to be taken into account when setting the receiver frequency and deciding what $f_A$ is.

Drift in receiver frequency between the two measurements will create an error in both sample rate and receiver frequency calibration. A drift of < 0.01Hz over the period required for the measurements is acceptable. With unstabilised crystal oscillators as receiver reference, and off-air frequency standards in the LF range, I have found that drift can stay within a few milli-hertz over several minutes, provided the equipment is given plenty of time to stabilise, is kept away from heat sources and rapidly changing ambient temperature, etc. Equipment with TCXO or OCXO reference oscillators is considerably better. It is a good idea to repeat the measurements a few times to make sure significant drift is not occurring.

**3) Limitations**

There are some significant limitations to the accuracy and application of this calibration technique:

The frequency calibration utility in WOLF does not appear to give correct results when the overall frequency error is greater than 1Hz. At 198kHz, a 1Hz error is about 5 parts per million, and at 60kHz it is about 17 parts per million, however calibration of most receivers will be within this. If in doubt, check the frequency offset determined by WOLF is consistent with the frequency of the carrier in the WOLF spectrum display.

This method cannot be used when the receiver has more than one independent oscillator determining the received frequency, since this introduces additional unknown variables. Examples of this are a receiver used with an LF – HF converter with its own crystal oscillator, receivers having a separate, tunable BFO, or some types of frequency synthesiser using a separate interpolation oscillator. However, the vast majority of modern receivers use a single reference oscillator.

Most modern synthesised receivers use DDS synthesisers. The tuning step of a DDS is a binary fraction of the reference frequency, i.e. it is not an exact whole number of hertz. Therefore, the actual output frequency does not normally correspond exactly to the indicated "dial" frequency, even when the reference oscillator is perfectly accurate; the tuning microcontroller selects the closest available frequency step. The actual tuning step is typically a few tens of millihertz, so errors of this order can be expected in frequency measurements, which will vary with the frequency setting. In principle, this error is predictable, but in practice prediction is not normally feasible because it requires knowledge of the algorithm used to calculate DDS phase accumulator values, as well as the reference frequency, frequency multiplication scheme, etc. Audio frequency measurement accuracy to a few tens of millihertz limits the accuracy of sample rate determination to a few tens of parts per million, however this is good enough for WOLF and most other things. The receive frequency calibration accuracy is of the order of 0.1 ppm, which again is good enough for most things, including WOLF.

## 4) Deriving the Formulae

The overall effect of the frequency conversions inside a radio operating in SSB mode are to produce an audio output frequency equal to the difference between the incoming standard signal frequency and the "carrier" frequency set on the receiver dial, i.e:

$$f_{Audio} = f_{SIG} - f_{Carrier} \text{ in USB mode, or}$$
$$f_{Audio} = -f_{SIG} + f_{Carrier} \text{ in LSB mode}$$

The measured value of $f_{Audio}$ is modified by two sources of error. The first source of error is due to the synthesiser reference frequency not being exactly equal to its nominal value; if the nominal reference frequency is $f_R$ and the actual reference frequency is $f_R'$, then the actual carrier frequency is $f_{Carrier}(f_R'/f_R)$. The ratio $(f_R'/f_R)$ is the same whatever frequency the receiver is set to. The absolute frequency error is therefore proportional to the set carrier frequency. (Note that this only applies to radios in which all conversion oscillator frequencies and the BFO frequency are derived from a common reference; this is true for the vast majority of modern synthesised receivers). The second source of error is the sound card sample rate. This results in the audio frequency $f_{Audio}'$ measured by the software being different from the actual audio frequency by an amount $f_{Audio}' = (f_s/f_s')\, f_{Audio}$, where $f_s$ is the nominal sample rate, and $f_s'$ the actual sample rate.

The measured audio frequencies obtained in USB and LSB modes can therefore be written as simultaneous equations:

$$f_{UA} = \left(\frac{f_s}{f_s'}\right)\left(f_{STD} - \left(\frac{f_r'}{f_r}\right)f_{USB}\right) \qquad (1)$$

$$f_{LA} = \left(\frac{f_s}{f_s'}\right)\left(-f_{STD} + \left(\frac{f_r'}{f_r}\right)f_{LSB}\right) \qquad (2)$$

Where:

- $f_s$ = nominal sample rate
- $f_s'$ =actual sample rate
- $f_r'/f_r$ = ratio of actual receiver carrier frequency to indicated frequency
- $f_{STD}$ = frequency of the off-air standard, e.g. 198.000kHz
- $f_{USB}$ = receiver carrier frequency setting in USB mode, e.g. 196.500kHz
- $f_{LSB}$ = receiver carrier frequency setting in LSB mode, e.g. 199.500kHz
- $f_{UA}$ = measured audio frequency output in USB mode (approx. 1500Hz)
- $f_{LA}$ = measured audio frequency output in LSB mode (approx. 1500Hz)

The unknown variables we wish to find are $f_s/f_s'$ and $f_r'/f_r$, which can be found by elimination or otherwise ("an exercise for the reader", as they used to say!) to be:

$$\left(\frac{f_r'}{f_r}\right) = \frac{f_{STD}\left(1 + \dfrac{f_{LA}}{f_{UA}}\right)}{\left(\dfrac{f_{LA}}{f_{UA}}\right)f_{USB} + f_{LSB}} \qquad (3)$$

$$\left(\frac{f_s}{f_s'}\right) = \frac{f_{LA}f_{USB} + f_{UA}f_{LSB}}{f_{STD}\left(f_{LSB} - f_{USB}\right)} \qquad (4)$$

The WOLF calibration utility outputs the offset between the measured audio frequency and the expected, nominal audio frequency, and the nominal frequency is the difference between the off-air standard and the receiver carrier frequencies. The result that needs to be entered into WOLF (and usually other software where sample rate calibration can be done) is the actual sample rate $f_s'$. Also we want to know what the actual audio frequency of the signal we are trying to receive will be when the receiver is set to some different receive frequency. To produce convenient formulae to do this, we can make the following substitutions in Equations 3 and 4:

$f_A$ = expected audio frequency (e.g. 1500Hz)
$f_{LO}$, $f_{UO}$ = offset in LSB and USB mode respectively measured with WOLF frequency check utility (a fraction of a hertz).

$f_{LA} = f_A + f_{LO}$,
$f_{UA} = f_A + f_{UO}$
$f_{USB} = f_{STD} - f_A$
$f_{LSB} = f_{STD} + f_A$

Then, after some more exercise for the reader, the formula for the actual sample rate becomes:

$$f_s' = \frac{f_s}{1+\left(\dfrac{f_{UO}+f_{LO}}{2f_A}\right)+\left(\dfrac{f_{UO}-f_{LO}}{2f_{STD}}\right)} \qquad (5)$$

The receiver carrier frequency error is:

$$\left(\frac{f_r'}{f_r}\right) = \frac{1}{1+\left(\dfrac{f_{UO}-f_{LO}}{2f_A+f_{UO}+f_{LO}}\right)\left(\dfrac{f_A}{f_{STD}}\right)}; \qquad \text{But, since} \left(\frac{f_{UO}-f_{LO}}{2f_A+f_{UO}+f_{LO}}\right)\left(\frac{f_A}{f_{STD}}\right) << 1,$$

$$\left(\frac{f_r'}{f_r}\right) \approx 1-\left(\frac{f_{UO}-f_{LO}}{2f_A+f_{UO}+f_{LO}}\right)\left(\frac{f_A}{f_{STD}}\right) \qquad (6)$$

For signals in the LF range, and audio frequency errors of a few hertz or less, the error involved in this approximation is unlikely to be more than a few parts per billion. A more useful quantity to know is usually the fractional error in frequency, $\Delta f_r/f_r$:

$$\frac{\Delta f_r}{f_r} = \frac{f_r'-f_r}{f_r} = \left(\frac{f_r'}{f_r}\right) - 1 \approx \left(\frac{f_{LO}-f_{UO}}{2f_A+f_{UO}+f_{LO}}\right)\left(\frac{f_A}{f_{STD}}\right) \qquad (7)$$

The actual audio frequency the wanted signal will produce with a particular setting of carrier frequency, assuming USB mode is used, is then:

$$f_{Audio} = f_{SIG} - \left(\frac{f_r'}{f}\right) f_{Carrier} = f_{SIG} - \left(\frac{\Delta f_r}{f_r}+1\right) f_{Carrier}$$

$$f_{Audio} = f_{SIG} - f_{Carrier} - \left(\frac{\Delta f_r}{f_r}\right) f_{Carrier} \qquad (8)$$